Polling Zones Planning Problem

Haluk Saffet Balkan, Nidai Kordal

***Abstract*— This paper introduces an application of the use of electromagnetic push-pull mechanism to the solution of mathematical programming problems. The algorithm used is similar to the methodology used to solve "Travelling Salesman Problem" using a population based algorithm simulating the push-pull mechanism of electromagnetic theory, which was originally applied to unconstrained optimization problems in continuous space. The charge (objective function value) of each solution point in feasible population applies attraction and repulsion force on the points in population and moves the objective function towards optimality, i.e. to point with highest pull force. Two different versions of TSP based EM mechanism is used to find optimum political districts, where districts formed must be compact and contiguous, while each zone have almost equal population. The algorithms are tested to redraw borders of the fifty polling zones of North Cyprus.**

***Index Terms*—electromagnetic push-pull mechanism, meta-heuristics, political zones, travelling salesman problem**

I. INTRODUCTION

Elections are undeniably one of the important instruments democracies and they have pronounced effects on the society. The selection of the *polling zones* is one of the important factors that may affect the outcome of an election. How the borders of a polling zones drawn should be objective. In other words, the borders of political zones should be drawn independent of creating an advantage or disadvantage to any political party or any political person. The following criteria are usually used for objectivity of political polling zones: One-Man-One-Vote Principle (i.e. every polling zone to have approximately same number of voters), compactness and being contiguous.

In some democracies, political zones are manipulated (gerrymandering) in order to favour certain political party. In order to stop such unethical practices, in literature, various and neutral procedures have been developed to draw the borders of polling zones, generally solving a single-objective or multi- objective optimization models.

The term "Gerrymandering" is a term coined for the deliberate change of the boundaries of polling zones to manipulate the result of elections. It is named after Elbridge *Gerry*, who created a salamander shaped political zone in which opposition voters were concentrated (packed) in few zones so that in neighbouring zones opposition is diluted.

In this paper, a polling zone plan will be acceptable if each zone generated is contiguous and has at most ±5% population deviation from the mean. Subject to these constraints the algorithm will search for compact shaped polling zones subject to constraints explained above.

In Section 2, the problem of drawing borders of political zones is described. Section 3 presents TSP based electromagnetic push-pull mechanism Section 4 contains the algorithms used to draw political zones, and the results of the numerical experiments for North Cyprus. Section 5 concludes the paper and state possible future work.

II. THE DETAILS OF THE POLLING ZONES MODEL

In summary, a plan to create polling zones is an allocation of indivisible population units into required number of contiguous polling zones so that each polling zone have compact shape, and the populations of all polling zones lie within plus or minus 5% of their mean . Then, the criteria can be summarized as:

- *Population Equality:* All polling zones should have almost equal populations for fair representation in the assembly;
- *Contiguous:* There should a unbroken path between any two geographical points within a zone which lies completely within that polling zone;
- *Geographically compact:* ideally circular in shape as opposed to a thin and long salamander shaped figure, most powerful criteria to prevent gerrymandering;
- Conformity to present administrative boundaries;
- Demographic characteristics of the population.

Polling zone planning problem, which is a linear 0-1 programming problem, is also a subset of "combinatorial set partitioning problems". In set partitioning models, a one-to-one assignment o nodes to contiguous sets are performed, based on node weights without violating the weight constraint of any set. General form of model is (1):

$$\left\{\begin{array}{l} Max.cx \\ \mid Subj.to \\ Ax = e^T, X_j = 0 \text{ or } 1, \text{ for } j = 1,\dots,n \end{array}\right\} \quad (1)$$

And A is an *m by n* binary matrix, e^T is a sum vector of size m, and c is a vector of n (arbitrary) real numbers (weights). In polling zone planning problem, regions must be divided into voting zones, where every citizen is a member of only one zone [2].

In recent years, meta-heuristic methods such as genetic algorithms, tabu search and simulated annealing has been applied to solve such problems. Ricca &Simone [3] had tested various algorithms including tabu search, descent, simulated annealing and stated that "old bachelor" acceptance produced the best results in the most of the test problems, especially when compactness criteria was used as the objective.

To solve the Polling zone Planning problem other fundamental physics theories have also been utilized for solving combinatorial problems: Potts model, a statistical physics approach [4]; and Spin system, which treats the problem by converting it to an energy function [5]. A meta-heuristic optimization has been developed based on Electromagnetism (EM), such that a charge is associated with points in a multi-dimensional solution space and each point will exert pull or push on other points, in process moving the objective to maximization. This process will start a “local search in Euclidian space in a population-based framework” [1].

Although, the contiguity and population limit criteria can be applied easily, design of compact zones is more complex. Hess, et al [6] used the squared distance of each person from the zone’s centre as a criterion. The fact that only the population units are chosen as zone centres also creates complications. Compactness measurement has a drawback because only population units can be zone centres. Garfinkel and Nemhauser [7] propose “distance” and “shape” perspectives to find geographical compactness. Marlin [8] shows that, formulation is appropriate when population centres are not fixed locations but it is not for fixed territorial centres. Young [9] showed that d^2_{ij} test is a measure of compactness, as well as a measure of size and, it will give a better measure of compactness for a small urban zone compared to a large rural zone of the same shape. Mehrotra, et al[10], uses a network approach to measure the proximity in terms of smallest number of transit population units between two given population units, s_{ij}, the number of edges in a shortest path from node i to j. The objective becomes minimization of the sum of distance cost of population units from the centre population unit of the zone. Chai, and Wei [11], used fuzzy set theory to generate more flexible plans for polling zones. Li, et al, [12] used compactness criterion in the objective function of a quadratic programming model.

The Polling Zones Problem, combinatorial problem, is classified as NP-complete problem and all the approaches listed above are heuristics. and exact procedures. In this study, a new heuristic approach is developed rather than using traditional mathematical programming techniques.

III. TSP BASED ELECTROMAGNETIC MECHANISM

Electromagnetic approach applies electromagnetic theory of physics. Points in the solution space are considered as electrically charged particles which move towards optimality using electromagnetic push-pull mechanism. The charge of each solution point is represented in the objective function value. Solution points with better objective function values have more charge than other points. As the application of mechanism, points with more charge (better objective function value) attract other points, and points with less charge (worse objective function value) repulse other points. As the charge value increases, size of attraction increases too. Finally, a total force vector exerted to a point is calculated by summing all push-pull force vectors for moving the point in the direction of total force applied on it [1].

The original electromagnetic approach applies on optimization problems with continuous variables in the following form, as given in eqs. (2) and (3).

$$\min f(x)$$
$$s.t. \quad x \in [L,U] \qquad (2)$$

[illegible] (3)

Birbil, et al [10] revised this formulation to ensure converge and proven that their revised version enables global convergence with probability one. Electromagnetic approach has been applied in various combinatorial optimization problems such as scheduling, travelling salesman and soft computing which also discuss hybrid applications.

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General scheme of Electromagnetic-like algorithm may be outlined as given below:

Initialize ()
iteration <= 1
  while iteration < *MAXNIT* do
    Local (*MLSI*, δ)
    F<= Calculate (F)
    Move (F)
    Iteration <= iteration + 1
end while

Algorithm has four phases Initialization of algorithm, computation of total force exerted on each particle, movement along the direction of the force and local search.

A. Initialization:

Algorithm starts with a randomly generated initial solution of *m* sample points in the feasible region defined by upper and lower boundaries of *n* dimensional hyper-cube. After that, the objective function value of each sample point is computed; *g(x)* and the point, which has the best objective function value, selected as best solution, x^b.

B. Local Search:

Local search is used to improve the current solution objective function towards a local optima. Upper and lower bounds in each dimension and random step parameter δ Є [0,1] are used to create a feasible random length to move sample points. In case neighborhood search finds an improved solution than the current point within *MLSI* iterations, update x_i as the x^b.

C. Calculate Force:

The charge of each point q_i, which determines point *i*’s pull or push power is calculated according to the objective function value of a point. Points with better objective function value have more charge than others. The charges are calculated using:

$$q^i = \exp = \left[-n \left(f(x^i) - f(x^b) / \sum_{k=1}^{m} \left[f(x^k) - f(x^{best}) \right] \right) \right], i = 1,2,..., m \qquad (4)$$

In calculating the force between two points, the Coulomb's law of the electromagnetic theory is used, where the force exerted on a point via other points is inversely proportional to the square of the distance between the points and directly proportional to the product of their charges. Point with better objective function value attracts other points, while a point with worse objective function value repels the others. Later a vector of the total force exerted on each point by other points is calculated. This vector will determine the movement direction of that point.

In the original Electromagnetic algorithm there is a possibility of being trapped by a local minima and thus the global optima may never be reached. Birbil, et al [13] introduced a perturbed point and changed the force computation, as a result, they show that algorithm converges global optimum with probability of one. The most distant point from the best point, x^b , is selected as the perturbed point x^p. The total force value of x^p amended, while others remain the same (5). In case of perturbed point x^p, the component forces are perturbed by a random number $0< \lambda <1$. The directions of the component forces are reversed according to:

$$F_j^p = \begin{cases} (x^j - x^p)\dfrac{\lambda q^p q^j}{\|x^j - x^p\|^2}, & \text{if } f(x^j) < f(x^p) \\ (x^p - x^j)\dfrac{\lambda q^p q^j}{\|x^j - x^p\|^2}, & \text{if } f(x^j) \geq f(x^p) \end{cases} \quad (5)$$

D. Move Point Along the Total Force:

Points are moved in the direction of total force vector by a random step length as in equation (6).

$$x^i = x^i + \lambda\left(F^i / \|F^i\|\right)(RNG),\ i = 1,2,\ldots,m \qquad (6)$$

$$RNG = \begin{cases} u_k - x_k^i, & \text{if } F_k^i > 0 \\ x_k^i - l_k, & \text{if } F_k^i \leq 0 \end{cases} \quad k = 1,2,\ldots,n \qquad (7)$$

Where u_k *and* l_k are the upper and the lower bounds of the k^{th} dimension and λ is the random step size .

IV. APPLICATION OF EMM FOR TSP TO POLLING ZONES MODEL

While originally Electromagnetic-like algorithm was applied to real-valued-solution problems, polling zone planning and Travelling Salesman (TSP) problems are integer programming problems. This study introduces two different approaches to model the polling zone planning as a TSP problem.

First is a method proposed by Debels, et al [14], for the scheduling problem given by Wu, et al [15], where ordered real numbers represent a tour. They use two different representations, either the activity-list (AL) or the random-key (RK). The difference between AL and RK forms of a schedule is given in Figure 2. A solution of 10-city TSP, where AL is used in electromagnetic-like method, is illustrated in table 2.

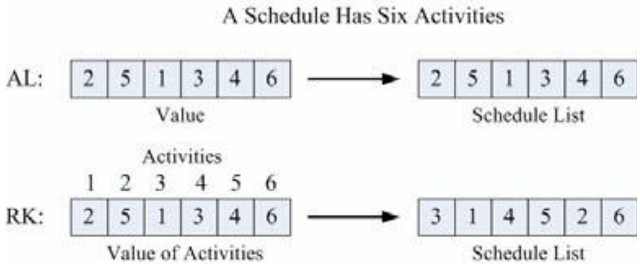


Figure 2 – Differences of AL and RK forms.

Table 1 – Illustration for use AL form in the Electromagnetic-like algorithm.

x_k^i	8.28	6.34	2.30	1.35	4.11	4.03	5.52	5.11	4.02	3.12
Initial Tour (City)	10	9	2	1	6	5	8	7	4	3
$\frac{F_k^i}{\|F^i\|}$	0.03	0.04	-0.04	-0.07	-0.05	-0.01	0.04	0.01	0.01	-0.01
$\Delta x_k^i = \lambda \frac{F_k^i}{\|F^i\|}(RNG)$	0.28	0.88	-0.30	-0.15	-0.92	-0.23	0.98	0.36	0.41	-0.09
$x_k^i \leftarrow x_k^i + \Delta x_k^i$	8.56	7.22	1.99	1.21	3.20	3.80	6.50	5.48	4.43	3.04
New Tour (City)	10	9	2	1	4	5	8	7	6	3

In the second method that was proposed by Kennedy, and Eberhardions, [17] for the discrete particle swarm optimization method discussed by Javadian, et al [16], is used. In this approach, a tour is represented by discrete-binary numbers. If a point i represents a tour of m cities as given below:

$X^i = (x_{11}^i, x_{12}^i, \ldots, x_{1m}^i, \ldots, x_{mm}^i)$, where $x_{nk}^i = 1$,

then in step n of the tour i city k is visited. In order to apply the binary tour representation, the total force vector calculated (as in Line 5 of Table 1) shows the chance of the binary variable taking the value one in the following step of the algorithm (Line 6). The greatest positive component of F^i is selected, and if the corresponding component of X^i is equal to zero, it is changed to 1. New Move procedure is applied to move the points and make necessary changes on X^i to obtain a feasible tour. Table 2 shows an application of these steps on an iteration of a tour of five cities with the sequence of 1, 3, 4, 2, 5 and the improved (moved point) tour when the greatest force in F^i is f^i_{33}.

A. Adaptation to the problem of Political zones Planning:

Above methods are used to find tours in TSP. This study is using activity-list representation for defining tours. Constructed tours are transformed into a polling zone plan by moving along the tour and summing the populations until the threshold population is reached, where the threshold defined as the "Ideal zone population" ± ε. Ideal zone population is obtained as the ratio of total number of voters to number of zones to be partitioned and ε = 5%. High Electoral Council of T.R.N.C. also requires ±5% tolerance limits for population parity, while generating a plan for polling zones.

Table 2 – Illustration of Binary form and movement.

Step of the tour	1	2	3	4	5
	1	0	0	0	0
	0	0	1	0	0
	0	0	0	1	0
	0	1	0	0	0
	0	0	0	0	1

⇨

1	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	1	0	0	0
0	0	0	0	1

The plan's population variance function (8) is based on ideal zone population, population penalty (P) and maximum tolerance allowed (Var $=_{5\%}$)

$$Measure_{POP} = \frac{P\sum_{i=1}^{n} Max(|Pop_i - IdDistPop| - Var_{5\%},\ 0)}{n * IdDistPop} + \frac{\sum_{i=1}^{n} |Pop_i - I}{n * IdD} \quad (8)$$

Compactness criterion is also used for evaluation of a polling zone plan. . As discussed before there are various alternatives for measuring compactness. In this study we are using a measure based on TSP. For each generated zone length of corresponding TSP tour ($Dist_i$) is calculated which is then normalized by the average distance travelled of all zones (*MeanTSP*) as shown by the equation (9).

$$Measure_{Comp} = \frac{\sum_{i=1}^{n} Dist_i}{n\,Mea} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (9)$$

Population and compactness measures are combined and the objective function value for the problem is obtained (10). In order to balance these two measures, θ is used.

$$ObjFuncVal = Measure_c + \theta * Measure_P \ \ldots\ldots\ldots \quad (10)$$

B. Parameters and Results

The primary parameters in this method for the sample solution given are listed in Table 3.

The algorithm is applied to the counties of T.R.N.C. by using the City and Administrative Maps of T.R.N.C. A special characteristic of Cyprus is its irregular shape which causes the algorithm to generate less compact zone plans.

Zoning plans generated for İskele and Lefkoşa counties are presented in figures 3 and 4, respectively. İskele county has a special shape. Zone plan of İskele, is not so reasonable because of the irregular shape of the Yeşilköy and Yenierenköy villages. These villages have too many neighbours as compared to the other villages, and they are located in the middle of the zone. Because of this reason, compactness measurement is low for the zone plan of İskele.

Table 4 - Parameters used in application of the method.

Name of Parameter	No. of sample points	Max. No. of iterations	Max. no. of local search iterations	Local search parameter	Population tolerance	Population penalty	Balance coef.
Symbol	m	*MAXNIT*	*MLSI*	δ	ε	P	θ
Value used	10	1000	100	1	0.05	100	50

Figure 3 - Zoning plan of İskele

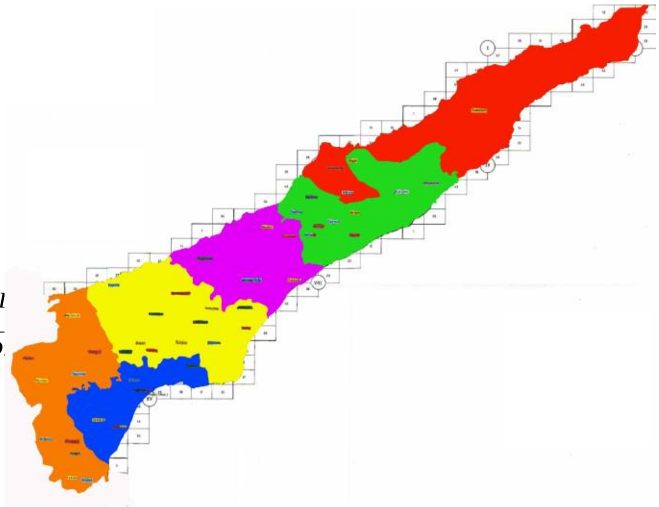
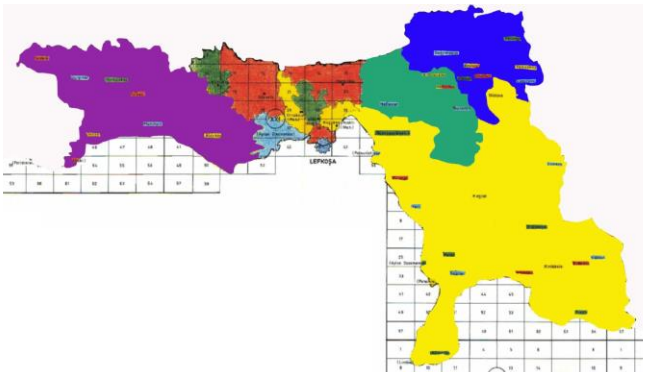


Figure 4 - Zoning plan of Lefkoşa



V. CONCLUSION

Electromagnetic-like algorithm presented in this paper uses TSP modelling and encoding to generate the polling zone plans. To our best knowledge, this is the first such algorithm used in this problem domain. The fact that compactness measure and behaviour of the objective function directly affects obtained zone plans, is a observed disadvantage.

In future the algorithm will be tested on benchmark problems for better comparison. To increase the quality of solutions, local search algorithms will be introduced in conjunction with multi-objective optimization and different computational methods.

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BIOGRAPHIES

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